Duality as a Central Organizing Theme in Linear Circuit Analysis Instruction

Or… The Yin and the Yang of Electrical Circuit Analysis?

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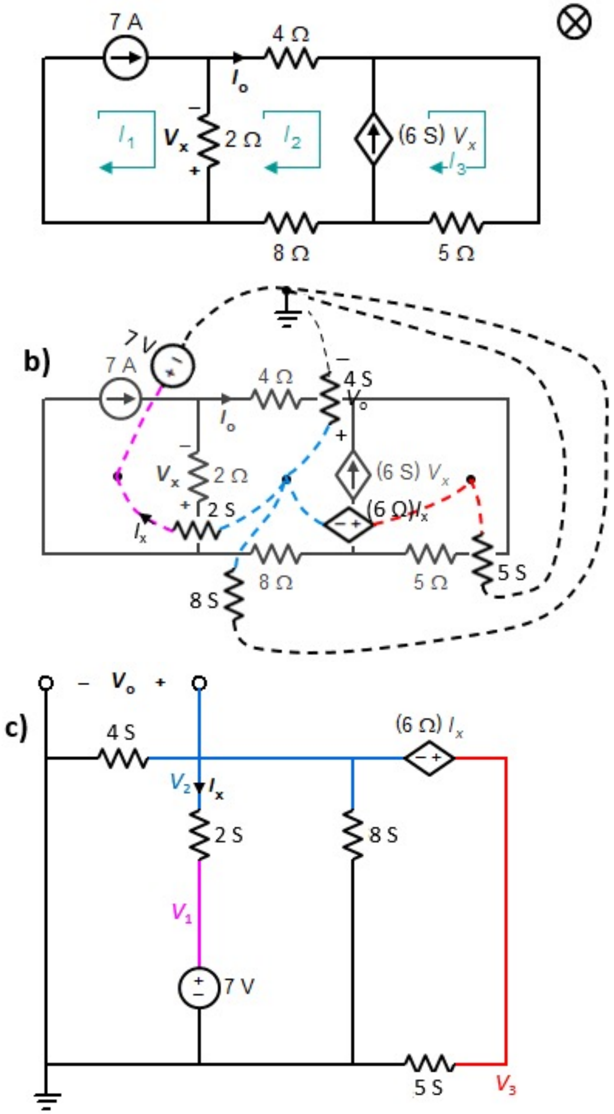
Introduction

- Circuit analysis is a core engineering topic
- The highly complex, interconnected nature of circuits imposes a high cognitive load on students
- Students must master many distinct ideas to achieve success
- Duality, a type of symmetry, can provide a unifying, overarching structure to the subject that can help promote learning (creates structural knowledge to organize declarative knowledge and facilitate recall from long-term memory)
- Duality is crucially important in many fields, from projective geometry (duality of lines and points) to Boolean algebra (duality of AND/OR operators and 1/0 literals). Textbooks on the latter nearly always exploit duality by presenting axioms and theorems as dual pairs
- Circuits exhibit strong duality, both in terms of geometric duals and lists of terms that can be interchanged in theorems about circuits, as first shown by A. Russell in *A Treatise on the Theory of Alternating Currents* (1904)

Sets of Dual Terms in Circuits

Branch current	Branch voltage
Kirchhoff’s current law (KCL)	Kirchhoff’s voltage law (KVL)
Mesh (including outer mesh)	Node
Mesh current	Node voltage/potential
Loop	Cutset
Parallel	Series
Current source	Voltage source
Open circuit	Short circuit
Resistance	Conductance
Inductance	Capacitance

Geometric Duals of Circuits



Current constraints: $I_3 - I_2 = (6\text{ S})\,V_x$, $I_1 = 7$ A

Sought current: $I_o = I_2$

$(I_2 - I_1)(2\ \Omega) + I_2(4\ \Omega) + I_3(5\ \Omega) + I_2(8\ \Omega) = 0$

(KVL for supermesh of meshes 2 & 3), and

$V_x = (I_2 - I_1)(2\ \Omega)$ (control voltage).

Voltage constraints: $V_3 - V_2 = (6\ \Omega)\,I_x$, $V_1 = 7$ V

Sought voltage: $V_o = V_2$

$(V_2 - V_1)(2\text{ S}) + V_2(4\text{ S}) + V_3(5\text{ S}) + V_2(8\text{ S}) = 0$

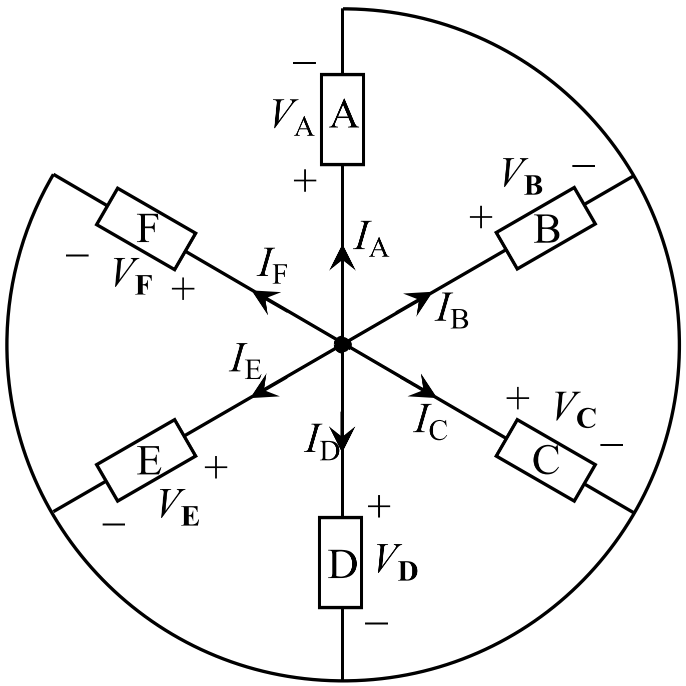
(KCL for supernode of nodes 2 & 3), and

$I_x = (V_2 - V_1)(2\text{ S})$ (control current).

Example of Duality as an Instructional Principle

Parallel Circuit

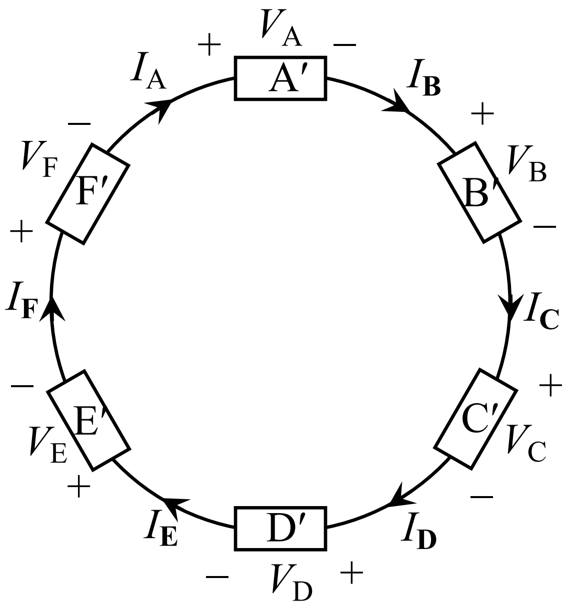
$$I_A + I_B + I_C + I_D + I_E + I_F = 0 \quad \text{(KCL)}$$



$$V_A = V_B = V_C = V_D = V_E = V_F$$

Series Circuit

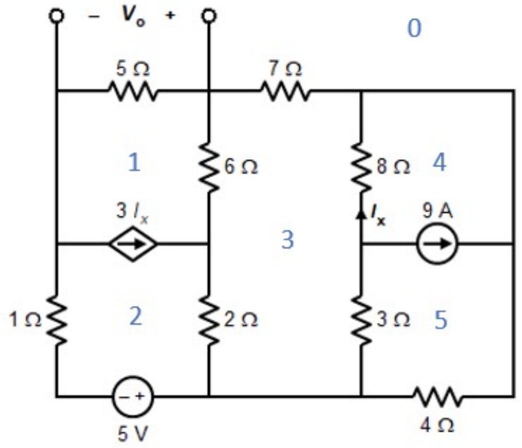
$$V_A + V_B + V_C + V_D + V_E + V_F = 0 \quad \text{(KVL)}$$

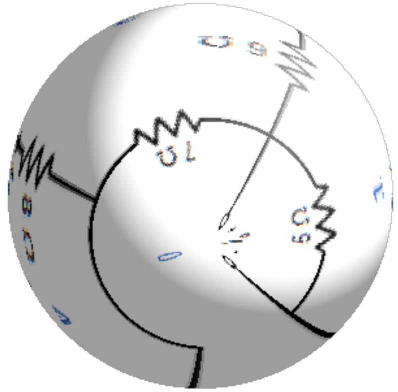


$$I_A = I_B = I_C = I_D = I_E = I_F$$

Why don't most textbooks use duality more extensively?

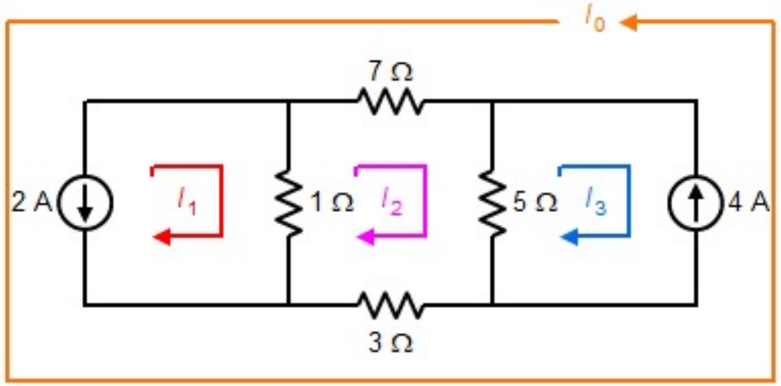
- Nodal and mesh analysis, two of the main techniques taught in circuit analysis, are not currently presented in a dual fashion!
- Nodal analysis starts by choosing a reference node; mesh analysis has no such step
- In nodal analysis, every element is connected to two nodes (one of which may be the reference node).
- In conventional mesh analysis, only "interior" elements are connected in two meshes, whereas "exterior" elements are seemingly connected in only one
- Further, the conventional definitions of series and parallel elements are not dual to one another (and the operational definition of series is seriously flawed)
- Fixing these problems can pave the way for full exploitation of duality!





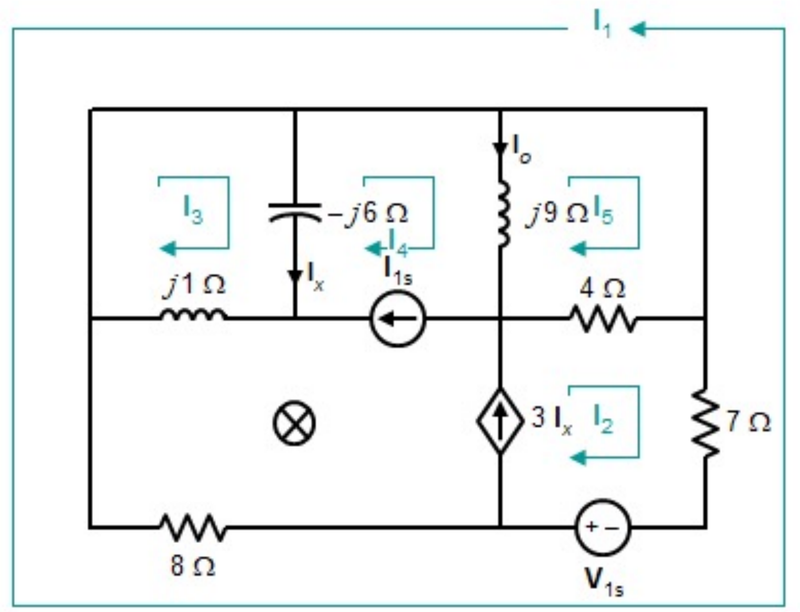
Which is the outer mesh now??

Dualizing Mesh Analysis



- Defining meshes inclusively: *A mesh is a loop that does not enclose any smaller loops, or that is not enclosed by or a portion of any larger loop in a planar circuit*
- The outer mesh must be defined as a true mesh to respect duality.
- A circuit can be turned "inside out" to make any mesh the outer mesh!
- Conventionally, one chooses a reference node in nodal analysis, but does not do so in mesh analysis. Implicitly, the outer mesh is always used as the reference mesh
- This need not be done! Any mesh can be the reference mesh (marked here with a "halt symbol," ⊗ analogous to a ground symbol). An intelligent choice may simplify the equations
- Further, this approach clarifies that *all* mesh currents are fictitious (not measurable); only branch currents (which are now always differences of mesh currents, just as all branch voltages are differences of fictitious node voltages) are measurable and real
- Changing the choice of reference mesh shifts the values of all mesh currents by the same amount, just as a difference choice of reference node shifts all node voltages by the same amount

Simplified Equations by an Improved Choice of Reference Mesh



- More importantly, nodal & mesh analysis are now exactly dual, as they should be!

With ref. mesh shown:

$\mathbf{I}_4 = 1\angle 120^o$ A, $\mathbf{I}_2 = 3\mathbf{I}_x$,
$\mathbf{I}_o = \mathbf{I}_4 - \mathbf{I}_5$, $\mathbf{I}_x = \mathbf{I}_3 - \mathbf{I}_4$,
$\mathbf{I}_3(j1\ \Omega) + (\mathbf{I}_3 - \mathbf{I}_4)(-j6\ \Omega) = 0$,
$(\mathbf{I}_5 - \mathbf{I}_4)(j9\ \Omega) + (\mathbf{I}_5 - \mathbf{I}_2)(4\ \Omega) = 0$, and
$\mathbf{I}_1(8\ \Omega) + 3\angle 0^o\ \text{V} + (\mathbf{I}_1 - \mathbf{I}_2)(7\ \Omega) = 0$.

With outer reference mesh:

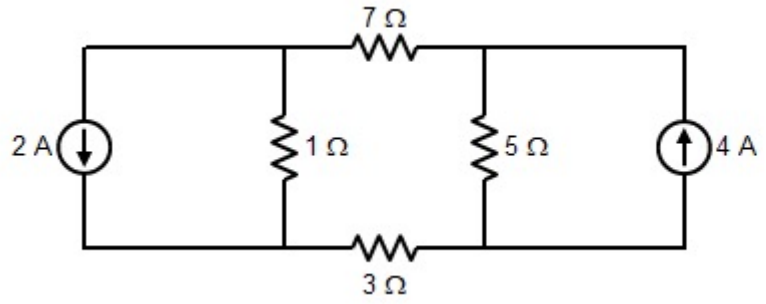
$\mathbf{I}_4 - \mathbf{I}_1 = 1\angle 120^o$ A, $\mathbf{I}_2 - \mathbf{I}_1 = 3\mathbf{I}_x$,
$\mathbf{I}_o = \mathbf{I}_4 - \mathbf{I}_5$, $\mathbf{I}_x = \mathbf{I}_3 - \mathbf{I}_4$,
$(\mathbf{I}_3 - \mathbf{I}_1)(j1\ \Omega) + (\mathbf{I}_3 - \mathbf{I}_4)(-j6\ \Omega) = 0$,
$(\mathbf{I}_5 - \mathbf{I}_4)(j9\ \Omega) + (\mathbf{I}_5 - \mathbf{I}_2)(4\ \Omega) = 0$, and
$(\mathbf{I}_1 - \mathbf{I}_3)(j1\ \Omega) + (\mathbf{I}_4 - \mathbf{I}_3)(-j6\ \Omega) +$
$(\mathbf{I}_4 - \mathbf{I}_5)(j9\ \Omega) + (\mathbf{I}_2 - \mathbf{I}_5)(4\ \Omega) + \mathbf{I}_2(7\ \Omega)$
$- 3\angle 0^o\ \text{V} + \mathbf{I}_1(8\ \Omega) = 0$

Defining Series Connections

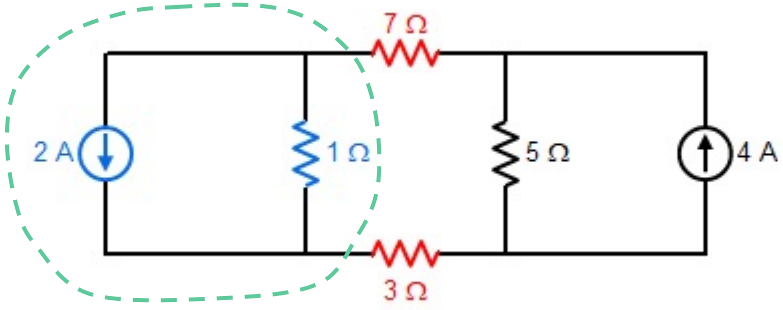
Conventionally, elements are in series if they carry the same physical current

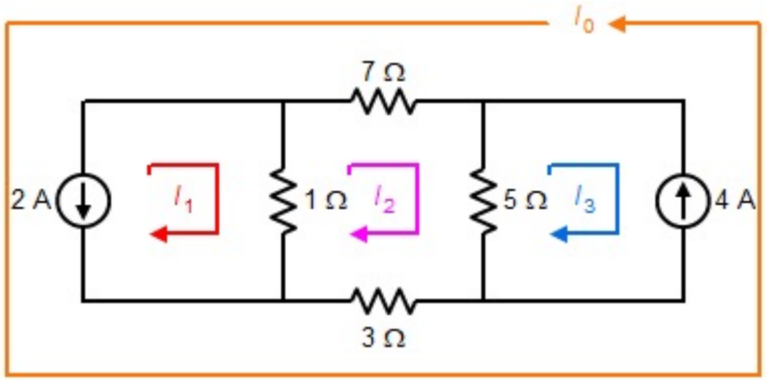
<u>Typical operational definition</u>: *Two elements are in series if they are connected to the same node, & no other elements are connected to that node*

Which, if any circuit elements at right are individually in series?



Typically, elements said to be in parallel if they are connected to the same pair of nodes, and therefore have the same physical voltage (= difference of node voltages)





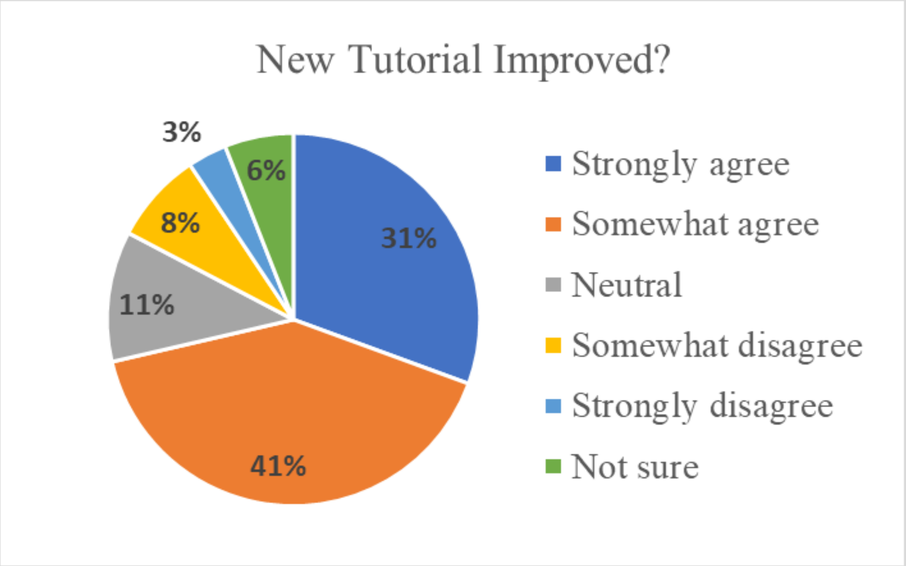
<u>Dual version of parallel definition</u>:
Elements are in series if they are connected in the same pair of meshes. (Branch currents are difference of same two mesh currents!)

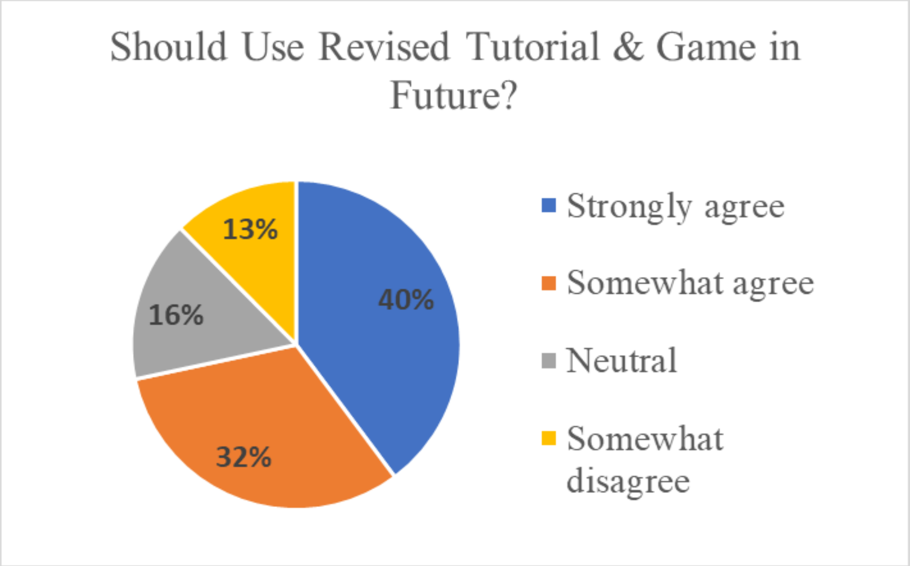
Implementing the New Dualized Approaches in Instruction

- Following were revised or created in Fall 2019-Spring 2020 to use the new dual approaches:
  - Lecture notes used by first author
  - Introductory multiple-choice tutorials in our step-based tutoring system (Circuit Tutor)
  - Corresponding exercises (“games”) in Circuit Tutor

Survey Results on New Approach to Series Connections (Spring 2020)

- 88 students in 3 sections used both versions of the Series-Parallel game
- 72% strongly or somewhat agree that introductory tutorial (using both mesh-based and chain-based definitions of series connections) is improved and should be used in the future; only 11-13% disagree
- Similar results obtained in Fall 2020





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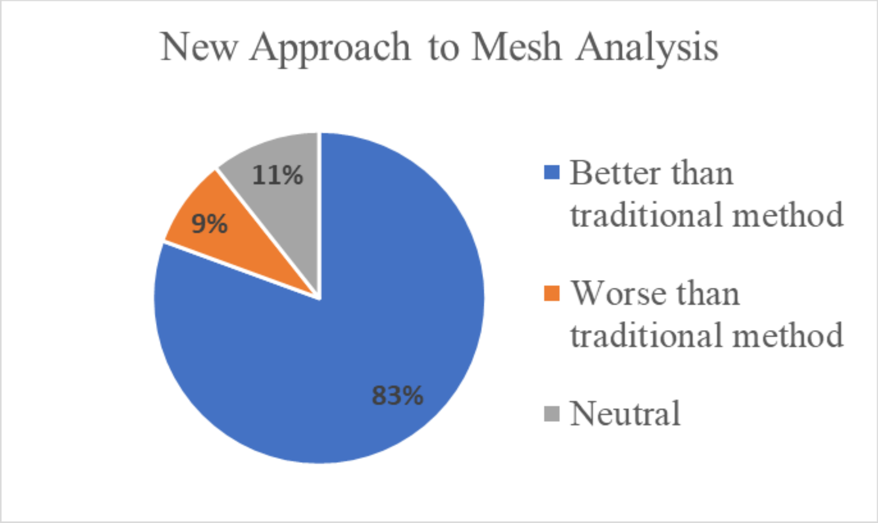
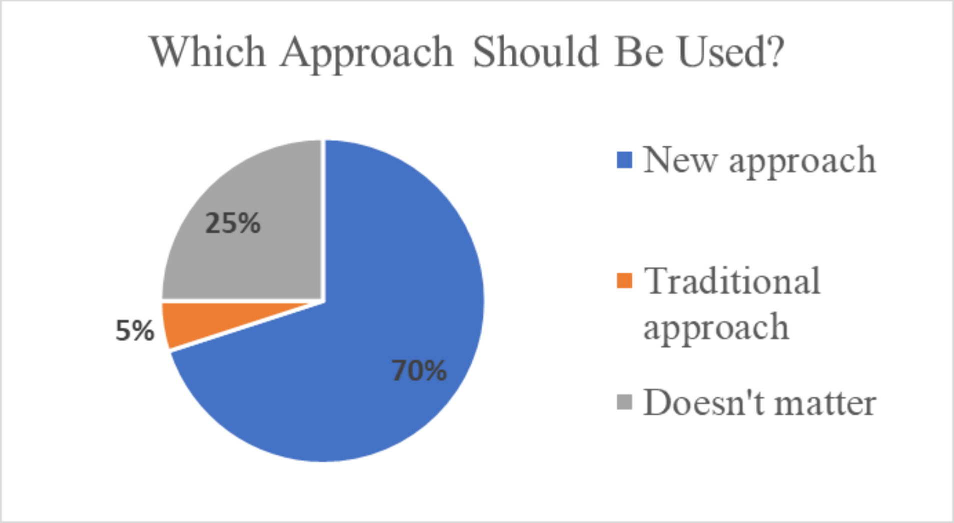
Survey Results on New Approach to Mesh Analysis (Fall ‘19, Fall ‘20, and Spring ’21)

- 195 students in 10 class sections at four institutions who used the revised version of the tutorial on mesh analysis (fully dual to nodal analysis)
- Comparing to traditional approach in their textbooks





Summary & Conclusions

- Duality is proposed as a unifying framework for linear circuit analysis instruction to enhance structural knowledge and minimize cognitive load
- Two key topics are not traditionally taught in a dual manner, namely nodal & mesh analysis, and series & parallel connections; our new approaches dualize both of these topics
- Dualized approaches were incorporated into a step-based tutoring system (Circuit Tutor) and used by up to 1800 students at 7 varied institutions taught by 24 instructors in 57 different class sections (F ’19-Sp ’21)
- Students favored the new, dualized approaches over the conventional ones by margins of 72-83% in various surveys
- Circuit analysis can be taught in a fully dualized fashion, and that students respond very positively to that approach
- The next step will be to develop a full set of instructional materials organized around duality

- Circuit Tutor is available for free use in courses; e-mail skromme@asu.edu for a free registration code

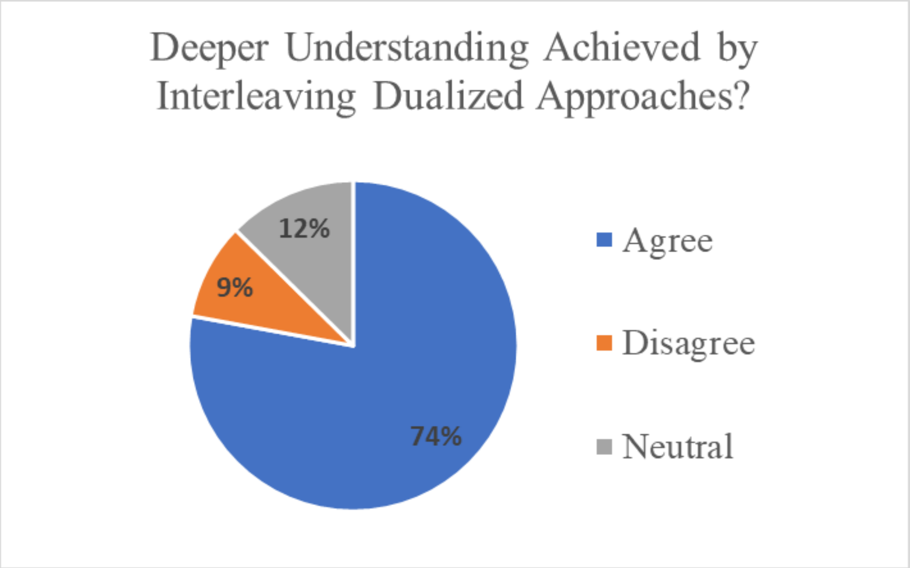
Spring 2021: Students Assigned to Interleave Dualized Nodal & Mesh Analysis

- Students in two sections were randomly assigned to either A) Complete Level 1 of both nodal & mesh analysis games in Circuit Tutor by a given date, then Level 2 by 2 days later, then Level 3 by 2 days later, then Level 4 by 3 days later; or B) Complete all the work by the same final due date with no requirement to interleave topics.

Sample Student Comments:

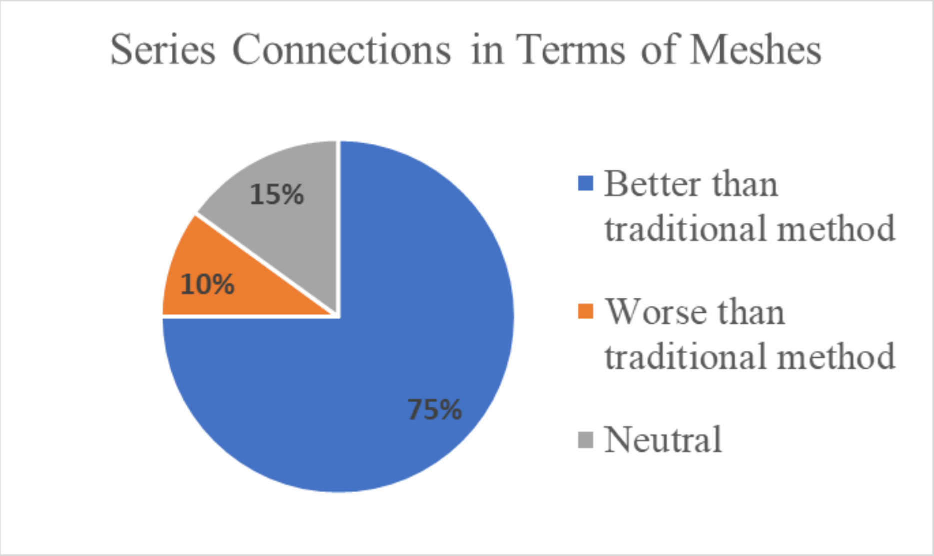
- The duality concept is genius. I think it is very important for students to understand and grasp the concept, even if it's a bit confusing (Which it isn't...)
- The new approach really helped me with mesh analysis because, although it confused me at first, it made things a lot easier once I got the hang of it. Especially since it was similar to the way we dealt with node analysis. I liked it.
- I never learned mesh analysis by textbook or the traditional method. However, getting to look at both nodal and mesh analysis through the same lens made it very easy to catch on. Since the same line of logic applies to both, it didn't feel like I was learning two separate topics, but rather two different applications of the same principles, which made a lot of sense to me.
- Tying mesh and nodal analysis together helped my comprehension and made retention easier
- The difference between working with currents and voltage is flipped. It is the same, but mirrored. I love that this new approach helped complete the symmetry of the two approaches.

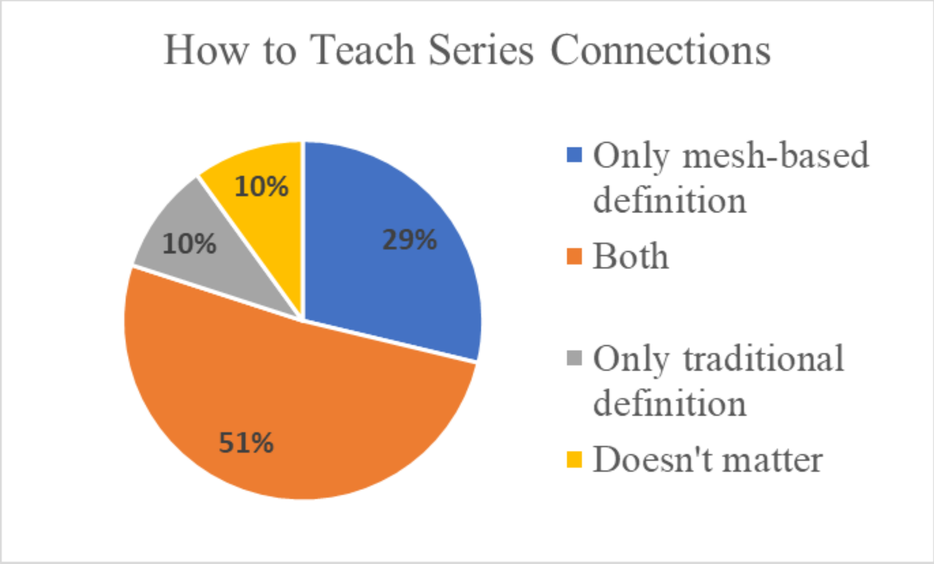
Survey Question For Those Who Interleaved:



Survey Results on New Approach to Series Connections (Fall 2020)

- 80 students at two institutions who used only the new version of the Series-Parallel game (ASU & University of Texas at El Paso)
- 75% thought the mesh-based definition is better; only 10% felt it was worse
- 51% favored using both definitions (as now done), 29% favored only the new definition, and only 10% wanted to stay with the old definition





Is Duality Currently Exploited in Linear Circuits Textbooks?

<u>No mention of concept</u>:

J. D. Irwin & R. M. Nelms, R. C. Dorf & J. A. Svoboda, A. Davis, F. T. Ulaby & M. M. Maharbiz, R. M. Mersereau & J. R. Jackson, S.A. Reza Zekavat, D. Bell (non-calculus level), A. Hambley (general EE text), R. A. Rohrer (1970)

<u>Mentions dual terms but not geometrically dual circuits</u>:

J.W. Nilsson & S. A. Reidel; R. E. Thomas, A. J. Rosa, & G. J. Toussaint; A. Agrawal & J. H. Lang (2005)

<u>Discusses dual terms and shows geometrically dual circuits</u>:

C. Alexander & M. Sadiku; W. H. Hayt, J. E. Kemmerly, & S. M. Durbin; A. B. Carlson; W. Y. Yang & S. C. Lee; M. E. Van Valkenburg (1955); E. A. Guillemin (1953); C. A. Desoer & E. S. Kuh (1969)

- Generally the principle is discussed to a limited degree in one or two sections, often only after introducing inductors & capacitors, due to their obvious duality.
- It has been used to motivate the idea of current sources and compare voltage & current dividers (Agrawal & Lang).

Is Duality Currently Exploited in Linear Circuits Textbooks? (cont.)

Desoer & Kuh (1969):

"The importance of duality cannot be overemphasized." (p. 455)

A. Guillemin (1953):

"Finally, the principle of duality may be mentioned as an important fundamental concept that should be prominent throughout the discussions comprising an introductory treatment of circuit theory. Here the term 'throughout' is used literally, since the principle of duality is not a topic that can effectively be disposed of by a concentrated discussion injected at some seemingly appropriate point, but instead is best dealt with by touching upon it again and again, bringing out each time some additional important aspect or application of this useful concept." (xii, Preface)

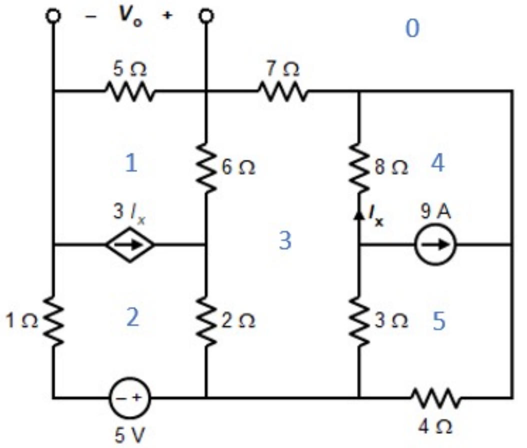
Thomas, Rosa, & Toussaint (2009):

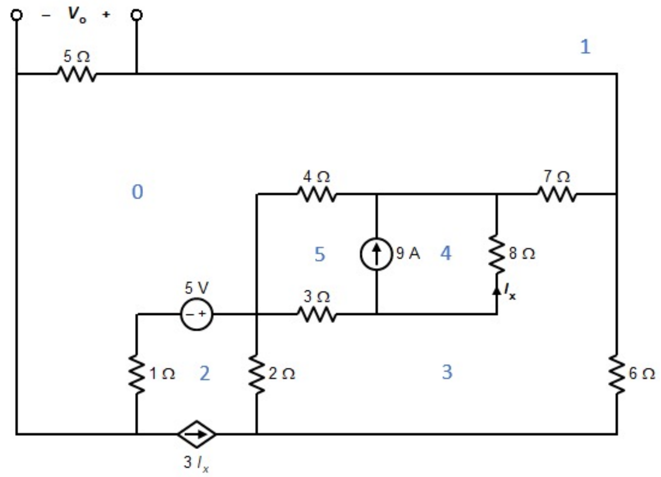
"This [duality] principle may help beginners gain confidence in their understanding of circuit analysis. When the concept in one column [of a table of dual quantities] is understood, the dual concept in the other column becomes easier to remember and apply." (p. 286)

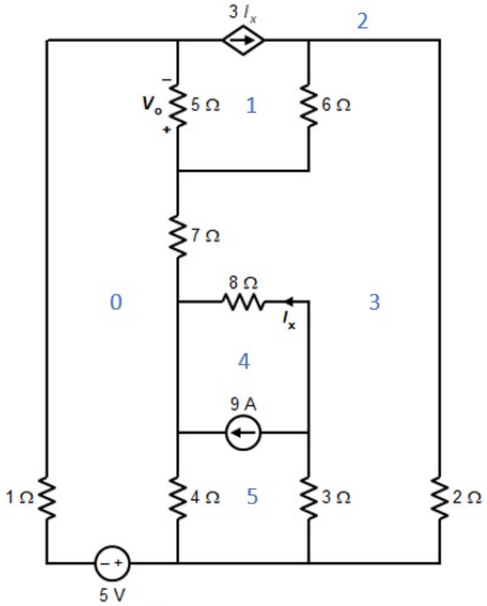
Conclusions (cont.)

- In our new approach to mesh analysis, the outer mesh is included as a real mesh and a reference mesh is selected, just as a reference node is selected in nodal analysis; the methods then become fully symmetrical
- An improved definition of series elements in terms of common meshes is given that is more inclusive than the usual one and that is properly dual to the normal definition of parallel elements (common nodes)

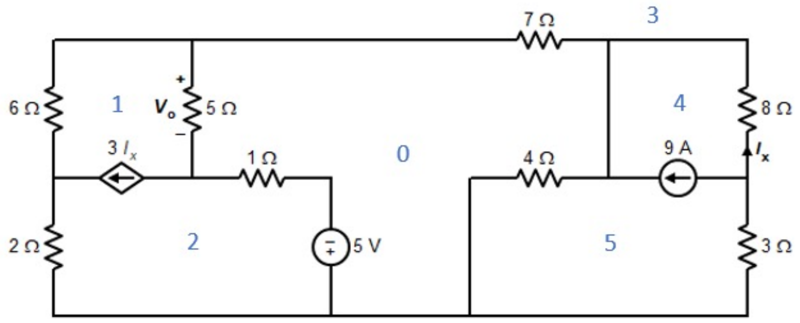
Turning a Circuit "Inside Out"

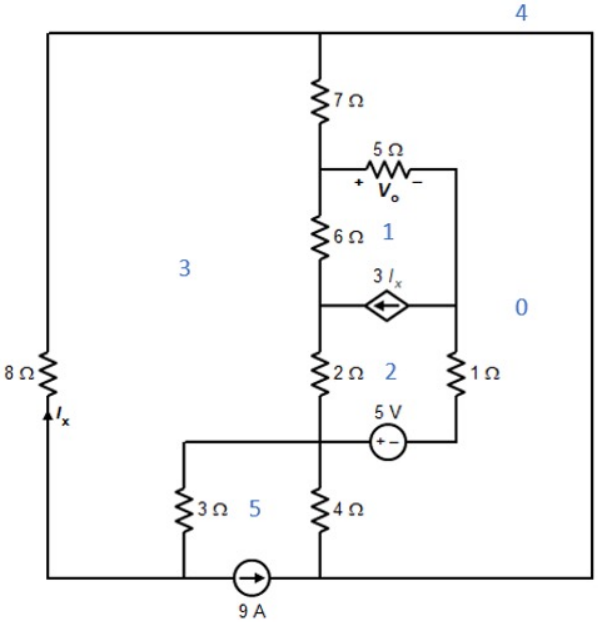


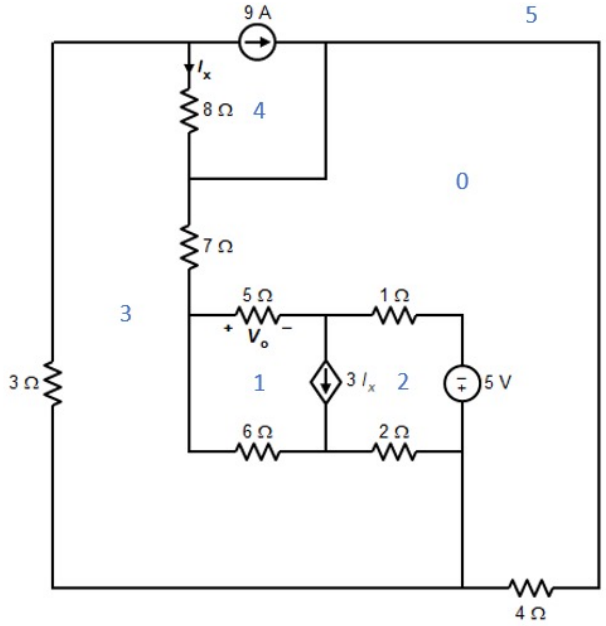




- Any mesh can be the outer mesh; the circuit is unchanged!
- Logically, "outer" & "inner" meshes must be treated on the same footing







Implementing the New Dualized Approaches in Instruction

- Lecture notes used by first author revised in Fall 2019 to reflect new approach to mesh analysis including explicit use of reference mesh and choosing it to advantage; used through present (Fall 2021)
- Introductory multiple-choice tutorial created on mesh analysis in Fall 2019 as part of our step-based tutoring system called Circuit Tutor, explaining the new approach; used in conjunction with four sets of step-based exercises ("games") on both AC and DC mesh analysis (equation writing & full solutions)
- Existing introductory tutorial on series & parallel connections in Circuit Tutor was revised in Spring 2020 to incorporate the new approach to series connections (in addition to the traditional, chain-based definition)
- The existing exercises ("game") on series-parallel connections in Circuit Tutor were revised in Spring 2020 to show colored mesh currents as an alternative to colored nodes, and to offer explanations in terms of common meshes for series elements