Addressing Barriers to Learning in Linear Circuit Analysis*

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Motivation

- Introductory linear circuit analysis is a foundational topic taught to many engineering majors (e.g., to over 600 students/yr. at ASU)
- A key concern is development of qualitative understanding of basic electrical concepts, and eliminating typical misconceptions, which has been studied previously only in the context of introductory physics courses
- A first goal is therefore to evaluate to what extent "typical" linear circuit instruction improves conceptual understanding of DC electricity, beyond that achieved in physics courses
- As we found this improvement to be poor, a second goal was to develop and evaluate instructional methods that can improve this understanding in a time-efficient way
- We further propose that a significant portion of the difficulties faced by students in these classes may be due to a failure to explicitly teach some of the principles of circuit analysis actually needed by students to solve problems ("The problems are nothing like the examples!")
- A third goal is therefore to enumerate some of these "missing" principles

Typical Student Misconceptions about DC Circuits

- A substantial literature about misunderstandings of (mainly) DC electrical circuits exists,* but there is little evidence that most engineering textbooks or instructors in this subject area are aware of these problems or address them
- Some typical misconceptions:
  - Regarding batteries as current sources rather than voltage sources
  - Believing that current is "consumed" as it travels through circuit elements
  - Believing that no voltage can exist across an open circuit (as opposed to no current through it)
  - Failure to identify series and parallel connections correctly (esp. the latter)
  - Not understanding the significance of short circuits
  - "Battery superposition" (the idea that more batteries deliver more power regardless of how they are connected)
  - A "sequential" model in which circuit elements only affect the current that is "downstream" from them

*See, e.g., P. V. Engelhardt and R. J. Beichner, Am. J. Phys. 72, 98 (2004); L. C. McDermott and E. H. van Zee, in Aspects of Understanding Electricity, edited by R. Duit, W. Jung, and C. von Rhoneck (Verlag, Schmidt, & Klaunig, Kiel, Germany, 1984).

Typical Student Misconceptions (cont.)

- Some such misconceptions may be related to ontological lateral misclassifications such as “electricity” being a substance rather than a process; these types of misconceptions are more resistant to instruction

  [M. T. H. Chi and R. D. Roscoe, in *Reconsidering Conceptual Change: Issues in Theory and Practice*, 2002, pp. 3-27; and M. Reiner et al., *Cognition and Instruction* **18**, 1 (2000)]
- Others have argued that these problems intead stem from “slippage between levels” (meaning confusion between agent-based microscopic levels involving electrons and the aggregate, macroscopic emergent phenomena such as current and voltage)

  [P. Sengupta & U. Wilensky, *Int. J. Comput. Math Learning* **14**, 21–50 (2009).]
- These misconceptions are typically robust and resistant to instruction
- An important open question is to what extent intensive “conventional” instruction in circuit analysis effectively addresses these misconceptions, as a fundamentally sound understanding of electricity should clearly be of importance to electrical (and other) engineers

Concept Inventory Testing Results

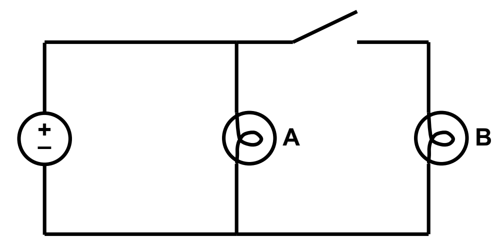
- We use the DIRECT 1.0 concept inventory developed and validated by Engelhardt & Beichner as a pre- and post-test to monitor learning of basic DC electrical concepts in our linear circuits course at ASU (EEE 202) (P. V. Engelhardt and R. J. Beichner, *Am. J. Phys*. **72**, 98-115, 2004)
- The pre- and post-tests were completed by 1287 and 990 students, respectively, in 20 sections from Summer 2012 through Summer 2014 with 14 different instructors
- Completing both pre- and post-tests generally counted as one homework assignment to ensure participation, but the scores did not affect students' grades (and students were aware of that fact)
- The average pre-test score was 49.4% (σ = 18.0%, N = 1287), similar to but slightly lower than the average found by Engelhardt & Beichner for university students of 53%.
- The average post-test for students whose instructors used conventional instruction was 57.2% (σ = 20.6%, N = 856), an increase of only about 8%
- Conventional, primarily quantitative instruction therefore has limited effectiveness in improving qualitative understanding
- We therefore worked to develop more effective instructional techniques

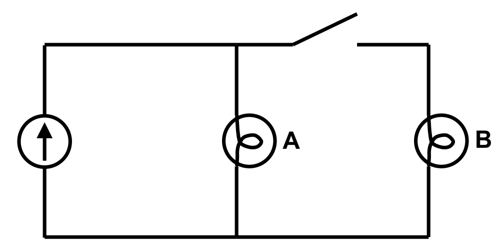
Addressing Misconceptions of Electrical Conduction

- Use microscopic models and analogies to explain conduction
  - Misconceptions such as current consumption, the idea that current is stored inside a battery, the idea that electric field inside a current-carrying conductor is zero (which probably originates from electrostatics training), and sequential models in which elements only affect current downstream from that element may be linked to a failure to appreciate the microscopic origins and natures of current and voltage
  - Circuits textbooks and most instructors rarely discuss microscopic models
  - We have presented a free electron (Drude) model for conduction emphasizing the microscopic physics (acceleration by **E**-field, scattering, mobility, etc.); similar to simulation approach of P. Sengupta & U. Wilensky (2009)
  - Need to emphasize macroscopic charge neutrality in nature (basis of Kirchoff's current law, etc.)
  - Helps students understand that **E**-field is present in a conductor, and that resistance does not vary with current or voltage (even though $R = V/I$!)
- Also present a rigid "ball in tube" model as a crude way to understand the conduction process; mobile charge always present, and same amount leaves one end as enters the other end (similar to hydraulic analogy)

Misconceptions of Voltage/Current Sources

- Comparing & contrasting current and voltage sources
  - Physics courses rarely treat current sources; focus on more common voltage sources (batteries, electrical outlets, etc.); never compare behavior
  - Students often think a bulb in parallel with a battery will dim when a second bulb is added in parallel (reasoning about current division, assuming battery supplies constant current)
  - Need to discuss examples like this with both current and voltage sources, comparing and contrasting behavior
  - Emphasize that voltage sources supply current and current sources supply voltage using a "feedback model," where an invisible operator adjusts the current of a voltage source to maintain a fixed voltage (or the voltage of a current source to maintain a fixed current)





  - Point out that sources must supply <u>both</u> current and voltage to supply power!
  - Get students to chant out loud in response to "What is the current through a voltage source?" that it is "whatever it needs to be" (to maintain its voltage) and similarly for the voltage of a current source

Misconceptions of Voltage/Current Sources (cont.)

- Emphasize that voltage sources create voltage <u>differences</u>, not absolute voltages on either end
- Emphasize that Ohm's law does not apply to voltage or current sources
- Directly address "battery superposition" misconception by discussing what happens when voltage sources are connected in series or in parallel (compare & contrast)

Misconceptions of Short & Open Circuits

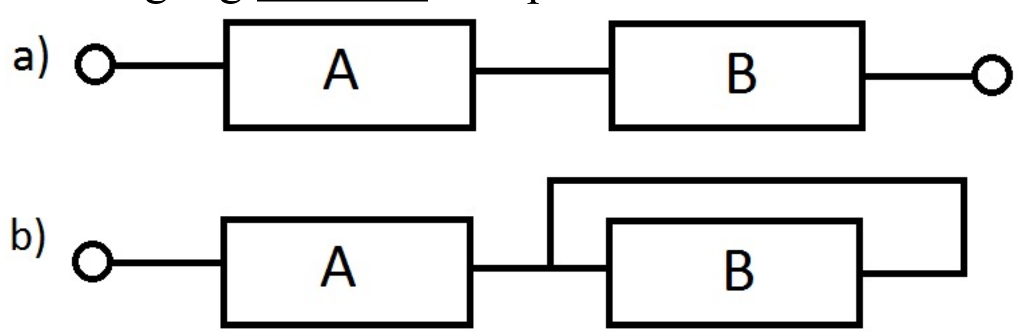
- Special case of voltage and current sources, respectively (with zero output)
- Short circuits have zero voltage, <u>not</u> zero current
- Open circuits have zero current, <u>not</u> zero voltage (very common misconception)
- Discuss as examples of Ohm's law, but with $R = 0$ or $R = \infty$, respectively
- Explicitly discuss behavior of shorted and "dangling" circuit elements
- Christmas light example (modern series design)

Misconceptions of Series & Parallel Connections

- Must give accurate definition of series connections
- Avoid confusion between geometrically parallel and electrically parallel elements
- Our interactive tutorial game is helpful on this topic
- Understanding the possible effects of <u>terminals</u> on series connections (parallel connections not affected)
  - Can be used to measure voltage, "view" input impedance, or connect a subcircuit (as when discussing Thévenin & Norton equivalents)
  - Latter two cases can change series relationships, voltage measurement does not
  - Distinction rarely emphasized in textbooks, but can cause confusion because terminals "look" the same in each case
  - We have developed an extension of our series-parallel tutorial to directly address this confusion, and to prepare students to evaluate input resistances and impedances correctly
  - Must explain exactly <u>how</u> input resistance is measured to appreciate the issue

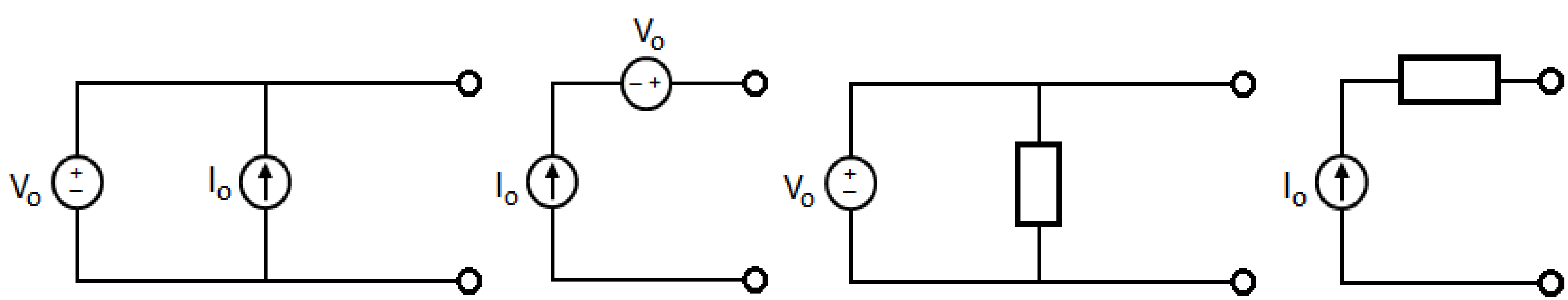
What Important Ideas About Circuit Analysis Are We Not Teaching (But Students May Need)?

- Circuit Topology: Hinged Circuits
  - Circuit is "hinged" if it can be drawn such that two subcircuits are connected by only a single wire
  - Equivalently, removing some node in the circuit leaves it disconnected
  - A hinged subcircuit can be either (a) dangling or (b) shorted
  - KCL implies that the current through the connecting wire must be zero, so two portions cannot influence each other (absent mutual inductance or dependent source variables)
  - Hinged circuits can be simplified by ignoring the portion that does contain the "sought quantities" (circuit variables we are trying to find)
  - Shorted and dangling elements are special cases of this situation



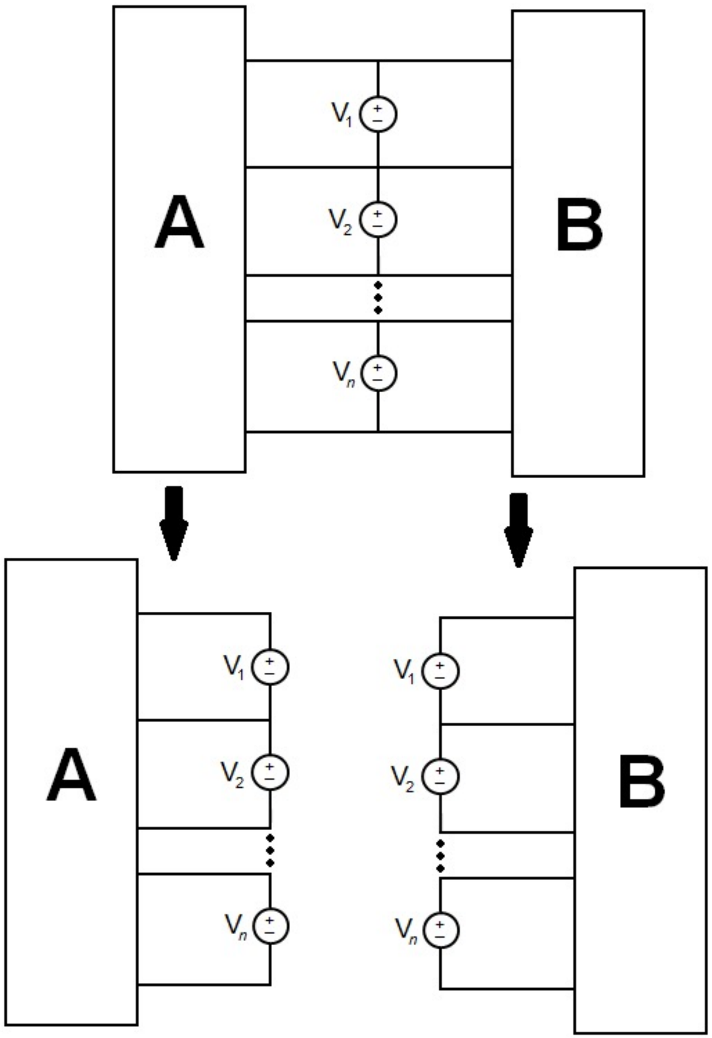
Ideas About Circuit Analysis (cont.)

- "Redundant" circuit elements
  - A current source in parallel with an ideal voltage source is redundant (i.e., has no effect on the rest of the circuit, other than modifying the current and power delivered by the voltage source)
  - Same comment applies to a voltage source in series with an ideal current source
  - Same ideas apply to a passive element in parallel with a voltage source or in series with a current source, or an arbitrary subcircuit
  - Any such elements can be removed to simplify a circuit unless the source power or current/voltage is sought



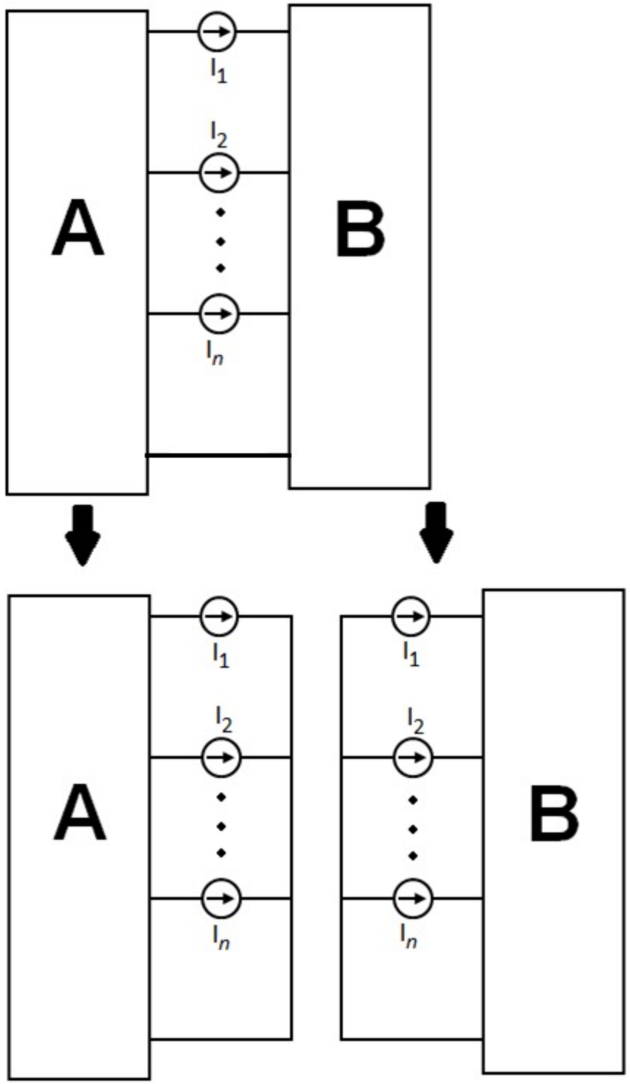
Ideas About Circuit Analysis (cont.)

- Voltage and current splittability
  - A circuit is *voltage splittable* if replacing all of the voltage sources in a chain by short circuits makes the circuit become hinged
  - It can be re-drawn as two separate circuits, each connected to the same set of sources (now duplicated)
  - Absent mutual inductance or dependent source linking, only the circuit with sought variables of interest need be solved (simplification method)
  - Redundant circuits are a special case where there is only one voltage source in the chain



Ideas About Circuit Analysis (cont.)

- Voltage and current splittability (cont.)
  - A circuit is *current splittable* if replacing some set of current sources (which all exit a closed surface) by open circuits results in the circuit being hinged
  - It can be re-drawn as two separate circuits, each connected to the same set of sources (now duplicated)
  - Absent mutual inductance or dependent source linking, only the circuit(s) with sought variables of interest need be solved (simplification method)
  - Redundant circuits are a special case where there is only one current source exiting the surface

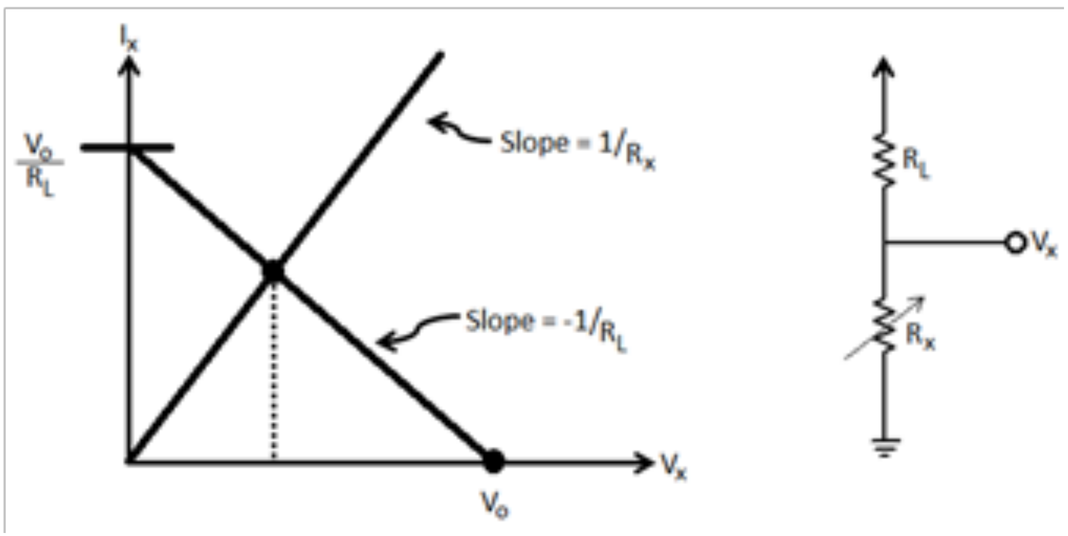


Ideas About Circuit Analysis (cont.)

- Replacement theorem
  - Once we have solved for a particular voltage or current in a circuit, we can replace the element having that voltage or current by an independent voltage source having the same value of voltage or by and independent current source having the same value of current
  - Doing so is useful in solving iterative problems or when finding initial conditions in a transient circuit

Applications to Voltage & Current Division

- Need to emphasize connection between voltage division and series connections, and between current division and parallel connections
- Can use a linear load line construction to illustrate voltage or current division graphically:



- Confusion can result from using resistance in discussing both voltage and current division (conductance is more natural in the latter case)

Applications to Voltage & Current Division (cont.)

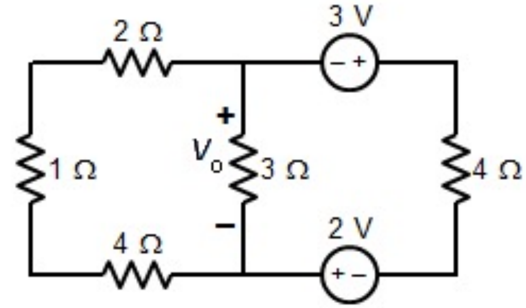
- Textbooks usually explain voltage and current division only for single loop or single node pair circuits, respectively
- These methods can however be used in several other cases:

  a) Simplifiable circuit: Reduces to single loop (or node pair) after combining elements in series and parallel (or even wye-delta transformations), or eliminating hinged elements or subcircuits

  b) Voltage splittable (redundant) circuits; simplify to single loop (or node pair) by discarding one portion of circuit

  c) Iteratively solvable circuits: Can initially combine elements in series or parallel, then use replacement theorem and reconstruct original circuit to solve

a)



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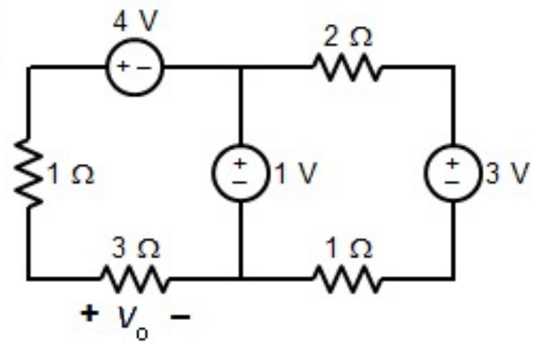

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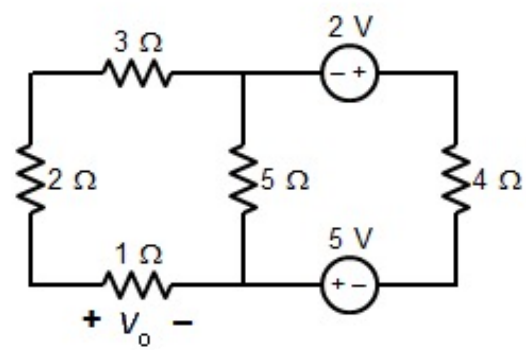

Can These New Approaches Improve Qualitative Understanding?

- I used the following approaches in my section of EEE 202 in Spring 2013:
  - Emphasized macroscopic charge neutrality
  - Presented the Drude model and ball-in-tube models of current flow
  - Emphasized that voltage sources establish voltage differences
  - Explicitly discussed the properties of open and short circuits
  - Discussed series & parallel connections carefully and had students complete the series-parallel exercise (though many other instructors also did the latter).
  - Discussed the hinged circuit concept
  - Emphasized the nature of voltage and current division
  - Differentiated the different roles of terminals in different cases
- In addition to the above, I used the following in Fall 2013:
  - Used the Christmas lights example to illustrate open circuits
  - Discussed control-loop models of independent current & voltage sources
  - Compared and contrasted current & voltage sources
  - Discussed redundant circuit elements (but not splittability)

Improving Qualitative Understanding (cont.)

- Methods used to introduce ideas:
  - Lecture on concepts (using white board, no PowerPoint)
  - Qualitative questions on homework and exams
  - Discussion in review sessions (in response to student questions)
- To test the effect, we compared concept inventory pre- and posttest data for my two sections (Fall & Spring 2013) with same data for 13 other instructors teaching the same course at ASU
- First, an ANOVA was conducted on the pretest scores to see if there was a difference among the instructors' classes. It showed a difference, $F(15, 1188) = 12.81$, MSE = 27.52, $p < 0.001$
- To statistically control for these pre-existing differences among the instructors' classes, an analysis of covariance (ANCOVA) was conducted on the posttest scores using the pretest scores as a covariate
- There was a statistically significant effect for instructor on the posttest scores, $F(15, 807) = 7.32$, MSE = 22.14, $p < 0.001$

Improving Qualitative Understanding (cont.)

Means (and standard deviations) for pretest and posttest scores

Instructor	*N*	Pretest	Posttest	Adjusted
1	18	14.1 (4.5)	15.8 (5.1)	15.8
2	25	13.4 (4.8)	18.1 (4.1)	18.6
3	17	17.1 (5.6)	18.6 (5.0)	17.0
4	18	16.2 (6.8)	19.2 (6.1)	17.8
5a	79	14.9 (5.2)	19.8 (5.5)	19.5
5b	65	14.5 (4.6)	22.4 (4.6)	22.3
6	67	12.6 (4.2)	16.0 (5.7)	16.9
7	103	13.3 (5.2)	15.7 (6.4)	16.3
8	27	16.0 (5.2)	18.2 (5.3)	16.9
9	41	18.9 (5.2)	18.8 (6.2)	16.0
10	38	14.6 (4.7)	16.3 (5.7)	16.7
11	29	13.3 (4.5)	15.9 (5.9)	16.6
12	65	15.8 (5.7)	17.0 (5.7)	16.0
14	79	13.0 (5.1)	15.9 (6.0)	16.4
15	64	14.6 (6.0)	17.4 (6.5)	17.0

Improving Qualitative Understanding (cont.)

- Post-hoc pairwise comparisons revealed that my students in Spring 2013 had significantly higher adjusted posttest scores than students who had instructors 1, 6, 7, 9, 10, 12, and 14, when adjusted for pretest score differences. (The adjusted posttest score was highest for instructor 5, but not with statistical significance for instructors 2, 3, 4, 8, 11, and 15)
- In Fall 2013, using more of the instructional strategies recommended here, my students had higher adjusted posttest scores (mean 22.3/29) than all other sections, including my students in Spring 2013 (mean 19.5/29)
- The effect size (comparing to all other instructors) was a Cohen *d*-value of **0.71 σ** for Spring 2013 and **1.02 σ** for Fall 2013 (large effects); raw score average in Fall 2013 was **77.4%**, vs. **57.1%** for all other instructors
- Thus, using these methods can result in significantly better conceptual learning, and using more methods improves the results
- Our plan is to try to incorporate these approaches into an interactive computer-based tutorial, so that other instructors can easily assign such work without having to heavily revise their lecture approaches

Conclusions

- Common misconceptions about electricity can be addressed using microscopic physical models of conduction, comparing and contrasting current & voltage sources and introducing "control-loop" models for them, explicitly discussing the properties of short and open circuits, by emphasizing the effects of terminals on series and parallel relationships, and using other methods discussed here
- The ability to solve circuits may be enhanced by introducing concepts of circuit topology not currently taught in most textbooks, such as hinged circuits, redundant sources and passive elements, and voltage and current splittability (this proposal remains to be tested empirically)
- DC circuit (DIRECT 1.0) concept inventory scores increased only from 49% to only 57% using conventional instruction in a one-semester circuits class, but up to 77% using methods described above (effect size of up to 1.02 σ)
- Plan is to incorporate these methods into an online tutorial that students can complete on their own, as part of our Circuit Tutor project